Asset Allocation of Sovereign Wealth Funds with Predictable Returns in Emerging and Imperfect Markets

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Keywords: Incomplete Market; Asset Income Predictability; Sovereign Wealth Funds; Asset Allocation

Abstract: In recent years, the global economy as a whole has been recovering steadily and moderately. The international market demand has been obviously improved, and the emerging market economy has been expanding, which provides a good investment opportunity for the global Sovereign Wealth Funds(SWFs). This paper mainly studies the strategic asset allocation and investment strategies of China's SWFs in emerging markets. Considering the predictability of asset returns, the dynamic asset allocation model of SWFs under incomplete market conditions is constructed. The incomplete market is transformed into a complete market by dimensionality reduction method. The results show that the imperfect market characteristics of stock assets above does not consider the prediction variables, the optimal allocation of asset allocation is different from the short-term and long-term investor, as the extension of investment horizon, risk assets such as equities allocation will also rise. In incomplete market, investors tend to be more conservative, and the proportion of investment in risk assets is significantly smaller than that in complete market. When the risk aversion coefficient decreases, the lower the risk acceptance rate is, the lower the proportion of investors in risk assets will be, and the corresponding final utility wealth value and the maximum expected utility wealth value will be smaller. Moreover, the liquidity index is negatively correlated with the return on equity assets, that is when the index declines, the return on equity increases.

1. Introduction

Ten years since the 2008 global financial crisis, the global economy still has not completely recovered from the aftershock of the crisis. The global economic expansion has slowed down sharply, the speculative investment in financial markets has increased obviously, and the globalization of trade has slowed down obviously. In this post-crisis era, the Sovereign Wealth Fund(SWFs)established in order to avoid exchange rate fluctuations and RMB appreciation, resulting in te loss of yield of single dollar bond assets, and to improve the use efficiency of foreign exchange reserves and the ability to maintain and increase the value of foreign exchange assets has become increasingly important. By the end of December 2017, the assets of China's SWFs had grown from \$249bn to more than \$930bn, making it the world's second-largest SWFs after the Norwegian government's global pension fund. In order to actively expand investment channels, construct more effective investment portfolios under effective risk control and obtain higher investment returns, SWFs should not only invest in mature markets of developed countries, but also increase the proportion of investment in emerging markets to diversify investment risks. Emerging market is an incomplete market. So far, classic portfolio models including Markowitz's singleperiod discrete mean-variance model have been discussed and established under the assumption of perfect market. The perfect market is an ideal investment environment, whose mathematical performance is that the number of basic assets available for trading in the security market is equal to the dimension of Brownian motion satisfied by the stock price, or all trading assets can be duplicated by the basic assets. The mature market of developed countries closer to full market and incomplete characteristics of emerging markets is more apparent, due to the interference of market friction factors such as transaction cost, in the imperfect market, the number of the underlying asset is less than the Brownian motion dimension, the underlying asset price follows a geometric Brownian motion with random variance and jump diffusion model, the asset price volatility has more "high-risk high-yield" features. Emerging markets have achieved market capitalisation growth of 33.9 percent so far in 2017, according to the MSCI regional index, significantly outpacing the expansion of about 17.6 percent in developed markets. By 2017, emerging markets accounted for 18% of the world stock market. Capital inflows contributed to a net increase of \$132 billion in the foreign exchange reserves of emerging market countries, making the market more stable in the face of future financial shocks. In the context of global economic recovery, emerging markets are more valuable for investment, so it is more practical for SWFs to seek asset allocation in such incomplete markets.

In an incomplete market, the undetermined equity of an asset cannot be duplicated by the underlying asset, and the market information state of this undetermined income can be observed. Campbell [1] found the predictability of stock premium through empirical test. Barberis [2] found that when asset income is predictable, long-term investors will increase the allocation proportion of stocks. Ang and Bekaert [3] found that short-term Treasury bond interest rate can effectively predict stock yield. In China, for example, Yang Chaojun et al. [4] think that liquidity factor can well predict stock asset returns in China. As for the asset allocation of incomplete market, Karatzas, Shreve, Xu [5] put forward the construction of virtual stock and construction of virtual complete market for investment; Zhang [6] proposed the compressed Brownian motion dimension method to construct the compressed complete market for investment. Sun Wanggui [7] described the dynamic asset portfolio of the two funds by determining the variance-optimal martingale measure and applying the dynamic asset parameters. Wang Xiuguo & Wang Yidong [8] studied the dynamic mean-variance portfolio based on random benchmark in incomplete market.

This article first to emerging market with mathematical description, not entirely the characteristics of application Zhang compression dimension reduction method is put forward by the imperfect market can be converted to full market, and consider the relationship between the returns and risk of power utility function of expectations, In order to establish the dynamic investment preference model under the optimal portfolio of power utility function in incomplete market. Considering the assets earnings predictability, into predictable factors in the model, and applies the martingale method to calculate the optimal investment strategy. Finally, the validity of the empirical model is illustrated by the representative emerging market countries--BRICS.

2. Model Construction and Solution

2.1. Mathematical Description of Incomplete Markets

Given a complete probability space (Ω, F, P) , N dimensional Brownian motion on (Ω, F, P) $W(t) = (W_1(t), W_2(t), \dots, W_n(t))^T$

The information flow generated in the domain is $F = \{F_t\}_{t \ge 0}$, Ω is the sample set, $\Omega \in F$, and F is closed, P represents the probability of each event $A \in F$. Let T > 0 be fixed investment term, $(\bullet)^T$ be matrix transpose, $\|\bullet\|$ be vector norm. and $E(\bullet)$ be mathematical expectation. Suppose that there are m+1 assets in the security market, Which are continuously traded within the interval [0,T], and one of them is risk-free asset, whose price is $P_0(t)$, and risk-free interest rate is r(t), satisfying the following ordinary differential equation:

$$\begin{cases} dP_0(t) = r(t)P_0(t)dt \\ P_0(0) = P_0 > 0 \end{cases}$$
(1)

The remaining m types of assets are risk assets such as stocks, whose prices are $P_1(t), P_2(t), ..., P_m(t)$, Which satisfies the following stochastic differential equation:

$$\begin{cases} dP_i(t) = P_i(t) \{ b_i(t) dt + \sum_{j=1}^n \sigma_{ij}(t) dW_j(t) \} \\ P_i(0) = P_i > 0 (i = 1, ..., m, m < n) \end{cases}$$
(2)

Where $b(t) = (b_1(t), b_2(t), ..., b_m(t))^T > 0$ is the return rate colume vector of risk assets, denoted by $\sigma(t) = \sigma_{ij}(t)_{m \times n}$ is the stock price Return matrix, and $\sigma(t)$ is the full rank matrix, that is, Rank $\sigma(t) = m$. Here, $r(t), b(t), \sigma(t)$ are measurable complex function on $\{F_t\}_{t\geq 0}$. It depends on the whole market before t time. $W_j(t)(j = 1, 2, ..., n)$ are independent Brownian movement. Under the above assumptions, the dollowing lemma can be obtained by restricting $m \le n$, Lemma 1:In a perfect market, $m = rank\sigma(t) = n$, that is ,the number of Brownian motion dimensions in risk assets in risk assets is equal to the number of securities assets; In a incomplete market, $m = rank\sigma(t) < n$, that is greater than the number of securities assets.

2.2. Optimal Wealth Value in an Incomplete Market

Let's say the market is incomplete and there's no dividend, no taxes, no transaction costs. The investor has the initial capital $x_0 > 0$, and at time t invests in the i(i = 1, ..., m) kinds capital proportion of risk assets is $\omega_i(t)$, and the proportion of investment in risk-free assets is $1 - \sum_{i=1}^{m} \omega_i(t), (i = 1, ..., m) \omega(t) = (\omega_1(t), \omega_2(t), ..., \omega_m(t))^T, t \in [0, T].$

Let X(t) be the wealth value owned by investors adopting self-financing strategy $\omega(t)$ at time t, then X(t) satisfies the following stochastic differential equation:

$$\begin{cases} dX(t) = X(t)r(t)dt + X(t)\omega(t)[B(t)dt + \sigma(t)dW(t)] \\ X(0) = x_0 \end{cases}$$
(3)

Among them

$$B(t) = \{b_1(t) - r(t), b_2(t) - r(t), ..., b_n(t) - r(t)\}^T$$
(4)

The market risk price in an in complete market is defined as:

$$\theta(t) = \sigma^T(t)(\sigma(t)\sigma^T(t))^{-1}B(t)$$
(5)

Lemma 2 Defines the discount process H(t):

$$H(t) = e^{-\int_0^t r(s)ds - \frac{1}{2} \int \|\theta(s)\|^2 - \int \theta^T(s)dW(s)}$$
(6)

 $H(\cdot)x(\cdot)$ is the martingale under the measure P.

Given the utility function U(x), investors expect to get the maximum expected utility of ending wealth as $E\{U(X)\}$, When $X(t) \ge 0$, $\omega(t)$ is a feasible investment strategy. Γ as a set of feasible investment strategies that meet the conditions, the optimal investment function under incomplete market can be obtained, that is, the maximum wealth value at the end of the period:

$$\max_{\omega(t)\in\Gamma} E\{U(X(t))\}, X \in \Gamma$$
(7)

2.3. Incomplete Market Transformation

The compression and dimensionality reduction method(Zhang,2007) is used to reduce the ndimensional Brownian motion of equation(2) to m-dimensional Brownian motion(m < n), so as to establish a completely compressed market. In this compressed perfect market, the number of risk assets, which satisfies the m-dimensional Brownian motion respectively, and the market is complete. Then the optimal investment in the imperfect market is transformed into the optimal investment function in the perfect market, and the optimal strategy of compressing the portfolio in the perfect market is also optimal in the imperfect market.

Firstly, reduce the dimension of Brownian motion and define the process:

$$V_{i}(t) = \sum_{j=1}^{n} \int_{0}^{t} \frac{\sigma_{ij}(s)}{\|\sigma_{i}(s)\|} dW_{j}(s), i = 1, ..., m.$$
(8)

Formula(2)can be written as:

$$dP_i(t) = P_i(t)\{b_i(t)dt + \|\sigma_i(t)\| dV_i(t), i = 1, 2, ..., m.$$
(9)

is the m dimensional random Brown movement that is satisfied with the stock price under incomplete market conditions. There is no mutual independence between $V_i(t)(i = 1, ..., m)$.

Then they construct the independent Brown movement. Where, if the yield matrix is denoted as $\hat{\sigma}(t)$. Because A(t) is not singular, $\hat{\sigma}^{-1}(t) = |\sum_{i=1}^{n} (t)A(t)|^{-1} = A^{-1}(t)\sum_{i=1}^{n} (t) \sum_{i=1}^{n} (t)$

$$\theta(t) = (\hat{\sigma}^T(t))^{-1} B(t) \tag{10}$$

And the price satisfaction of m kinds of stocks:

$$\begin{cases} dP_i(t) = P_i(t) \{ b_i(t) dt + \sum_{j=1}^n \hat{\sigma}_{ij}(t) d\hat{W}_j(t) \} \\ P_i(0) = P_i, i = 1, ..., m. \end{cases}$$
(11)

Among them, $m < n, \hat{W}(t) = (\hat{W}_1(t), \hat{W}_2(t), ..., \hat{W}_n(t))^T$ are the m Brownian movement in the probability space (Ω, F, P) , and the $\hat{W}_j(t)$, j = 1, ..., m are independent. Dynamic multi-stage investment model under incomplete market. At time T, the SWF investors conduct multi-stage asset allocation with an investment term of, and simply choose two kinds assets: risky assets(such as stocks) and risk-free assets(such as bonds). Assuming that the real interest rate of risk-free assets is constant r_j . The initial assets are $X_T = 1$, and the weight of stock assets in the portfolio is ω , then the ending assets are:

$$X_{T+t} = (1-\omega)e^{r_f t} + \omega e^{r_f t + r_{T+1} + \dots + r_{T+t}}$$
(12)

Where, r_{T+t} represent the continuous compound excess return rate of stock assets in T + t period and assume the cumulative excess return rate of stock assets in the whole investment period. The state variable is

$$R_{T+t} = r_{T+1} + \dots + r_{T+t} \tag{13}$$

According to(13), the optimal objective function of return of return for investors in incomplete market is:

$$\max E\{U(X_{t})\} = \frac{1-\gamma}{\gamma} [(1-\omega)e^{r_{f}t} + \omega e^{R_{T+t}}]^{\frac{\gamma}{\gamma-1}}$$
(14)

Where, γ represents the risk aversion factor.

Assume R_t is the state variable of asset income, X_{t-1} is the matrix of predicted dependent variables, α is the constant, β is the regression coefficient matrix, ε_t β is the erroritems. The establishment of prediction model can estimate the cumulative excess return rate of stock assets:

$$\mathbf{R}_{t} = \alpha + \beta X_{t-1} + \varepsilon_{t} \tag{15}$$

Where, $R_t = (r_t, x_t), X_t = \{x_{1t}, x_{2t}, ..., x_{nt}\}, \mathcal{E}_t \sim N(0, \Sigma).$

Here, r represents the excess continuous compound rate of return in the first period. The set of

prediction variables $x_t = (x_{1t}, x_{2t}, ..., x_{nt})$ is composed of various variables that have the ability to predict the return on assets.

Theorem 1 The dynamic asset allocation model considering asset predictable factors under the condition of incomplete market is as follows:

$$\begin{cases} \max E\{U(X_{i})\} = \frac{1-\gamma}{\gamma} [(1-\omega)e^{r_{f}t} + \omega e^{R_{T+i}}]^{\frac{\gamma}{\gamma-1}}, X \in \Gamma \\ R_{i} = \alpha + BX_{i-1} + \varepsilon_{i}, X(0) = x_{0}, X(t) \ge 0, \end{cases}$$
(16)

Given an $\hat{\lambda} > 0$, if (\hat{X}, \hat{Q}) is a martingale of (11), then the optimal solution can be obtained by using the martingale method

$$X^{*} = \frac{\gamma x_{0}}{1 - \gamma} \{ \exp(-\int_{0}^{t} r(s) ds - \frac{1}{2} \int_{0}^{t} \left\| \theta(s) \right\|^{2} ds - \int_{0}^{t} \theta^{T}(s) d\hat{W}(s) \}^{\frac{\gamma}{1 - \gamma}}$$
(17)

Acording to Lemma 2, Theorem 1 leads to Theorem 2:

Theorem 2 Under incomplete market conditions, when the price of risky assets at time t satisfies(1)and(2),risk-free interest rate r(t), risky assets cumulative excess return R(t) and return matrix $\sigma(t)$ are measurable deterministic functions of time, the curtain utility function of asset predictability is considered in the sense of dynamic optimal investment strategy $\pi^*(t)$, terminal wealth expectation $E[X^*(t)]$ and maximum expected utility value $E[U[X^*(t)]]$ are:

$$\pi^{*}(t) = \frac{\frac{1}{2\gamma} e^{-\int_{t}^{t} r(s)ds} - X^{*}(t)}{X^{*}(t)} [\sigma(t)\sigma^{T}(t)]^{-1}R(t)$$

$$E[X^{*}(t)] = x_{0}H(t)^{\frac{\gamma}{1-\gamma}} = x_{0}\{\exp(-\int_{0}^{t} r(s)ds - \frac{1}{2}\int_{0}^{t} \left\|\theta(s)\right\|^{2}ds - \int_{0}^{t} \theta^{T}(s)d\hat{W}(s)\}^{\frac{\gamma}{1-\gamma}}$$

$$E[U[X^{*}(t)]] = \frac{1}{\gamma} [\frac{x_{0}(1-\gamma)}{\gamma}]^{\gamma}H(t)^{\frac{-\gamma^{3}}{(1-\gamma)^{2}}}$$

$$= \frac{1}{\gamma} [\frac{x_{0}(1-\gamma)}{\gamma}]^{\gamma} \{\exp(-\int_{0}^{t} r(s)ds - \frac{1}{2}\int_{0}^{t} \left\|\theta(s)\right\|^{2}ds - \int_{0}^{t} \theta^{T}(s)d\hat{W}(s)\}^{\frac{-\gamma^{3}}{(1-\gamma)^{2}}}$$
(18)

3. Empirical Analysis of the Model

3.1. Data Selection and Processing

The stock market of the new developing countries is an incomplete market, and the whole market is prone to the phenomenon of risk fluctuation gathering. If the daily rate of return is taken as the sample to investigate, it is susceptible to the influence of market microstructure such as transaction spread and price. If the monthly rate of return is taken as the sample interval to investigate, the interval is too long, it may not fully reflect the real characteristics of stock market returns. Stock assets, therefore, r_t selectionon January 1,2008 to May 31,2018,the BRICS countries outside China in Brazil, Russia, India, South Africa, four countries in the MSCI index, the four countries of the 2720 samples data by day 5 days weeks for an average of 544 sample data, the data for a total of 2176 weeks(data from the MSCI Stock.Q).

3.2. Selection of Indicators of Prediction Variables

The predictive variable x_i of stock asset income, choose the single variable of liquidity factor. Liquidity factor is generally regarded as the ability to make and complete large-scale transactions quickly without changing prices or making small price changes, it has three characteristics: small price changes, large volume and fast trading hours. The smaller the price change caused by the completion of a large volume in a small period of time, the more liquid the market will be. But

fluidity is a difficult indicator to describe, because it cannot be directly observed and measured by simple methods. The comprehensive liquidity measurement index of the structure of Yang Chaojun et al.(2007) is adopted here:

$$IL_{t} = \frac{\left|r_{t}\right|}{V_{t}} = \frac{\left|\ln\left(\frac{P_{tc}}{P_{to}}\right)\right|}{V_{t}}$$
(19)

Where $r_t = \ln(\frac{P_{tc}}{P_{t0}})$ represents the composite continuous rate of return(take the logarithm of the rate

of return) of stock assets on the first day. P_{tc} is the closing price on the 1st day. P_{t0} is the opening price on the 1st day. V_t is the transaction amount of stock assets on the 1st day. And the unit is 100 million.

Example of asset allocation

Table 1 shows the results of the mean value of the posterior distribution of the parameter values of historical data. The values in brackets are standard deviations.

From January 2	008 to May 2018			
α	β			
-0.0082(0.0011)	0.9717(0.0013)			
0.0063(0.0052)	0.5871(0.0743)			
Σ				
0.0050(0.0007)	-0.2988(0.0682)			

Table 1 Parameter estimates of the prediction model of incomplete securities market return rate

It can be seen from table 1 that the average prediction coefficient is 0.9717, so the liquidity index has a good ability to predict the rate of return on equity assets, and the standard deviation is small (0.0013). The excess return rate of stock assets r_t is negatively correlated with the liquidity index $x_t = AIL_t$, and the correlation value is-0.2988, significantly different from 0. Since the state variable R_t of the expected return rate distribution of stock assets depends on the initial observation value of the liquidity indicator x_t , the median of the indicators during the selection period $x_T = 0.0132$.

If SWFs institutional investors have initial capital $x_0 = 1$ (Unit: ten thousand yuan), and invest in the markets of Russia, Brazil and India, assuming that the risk-free interest rate is r(t) = 0.05, the cumulative excess returns at the end of the three stock markets from January 2008 to May 2018 are calculated on a weekly basis $b(t) = \{0.18, 0.25, 0.33\}$, respectively. The fixed investment period is

T = 1 year, and the utility function is $U(x) = \frac{X^{\gamma}}{\gamma}$, where γ is the risk aversion factor, and the yield

	1.28	0.09	0.29		
matrix is:	$\hat{\sigma}(t) =$	0.09	2.29	0.34	
		0.29	0.34	3.01	

At this point, the marketis complete. And $\theta(t) = \{0.08, 0.07, 0.08\}^T$, $\|\theta(t)\| = 0.02$, by theorem 2 can get the optimal investment strategy are: $X^*(t) = \{0.0172, 0.0271, 0.0561\}^T$,

If $t = 0, \gamma = (2 \times 10^5)^{-1}, E[X^*(T)] = 12067.40, E[U[X^*(T)]] = 10655.80$ (Unit: ten thousand yuan), the optimal investment ratio of stocks in the Russian, Brazilian and Indian markets is : $\pi^*(t) = \{0.1053, 0.1645, 0.3440\}^T$, That is, Russia10.53%, Brazi16.45%, India 34.40%. The investment proportion of risk-free assets (bonds, etc.) is 38.42%. When the market is incomplete, the stocks of Brazil and India with higher yields are selected. It is still assumed that the risk-free interest rate is r(t) = 0.05, $b(t) = \{0.25, 0.33\}$, and return matrix is: $\hat{\sigma}(t) = \begin{bmatrix} 0.092.290.34\\ 0.290.343.01 \end{bmatrix}$

 $\theta(t) = A^{-1}(t) \sum_{t=1}^{T} (t)B(t) = \{0.07, 0.09\}^T, \|\theta(t)\| = 0.01$, also can get the optimal investment strategy are: $X^*(t) = \{0.0245, 0.0289\}^T$

If $t = 0, \gamma = (2 \times 10^5)^{-1}, E[X^*(T)] = 11637.70, E[U[X^*(T)]] = 10463.50$ (unit: ten thousand yuan), the optimal investment ratio of stocks in Brazil and India is: $\pi^*(t) = \{0.2088, 0.2461\}^T$,

That is 20.88% in Brazil and 24.61% in India. At this time, the investment proportion of risk-free assets (bonds, etc.) is 54.51%.

If $t = 0, \gamma = (1.5 \times 10^5)^{-1}$, $E[X^*(T)] = 11323.40, E[U[X^*(T)]] = 10124.50$ (unit: ten thousand yuan), Brazil and India's stock market, the optimal investment proportion for: $\pi^*(t) = \{0.1505, 0.1773\}^T$,

That is 15.05% in Brazil and 17.73% in India. At this time, the investment proportion of risk-free assets (bonds, etc.) is 67.22%.

4. Conclusion

From theoretical analysis and empirical test results, the following conclusions can be drawn:

In incomplete markets, the investment style is more conservative, and investors tend to invest in risk-free assets. The proportion of investment is smaller than that in the perfect market, which fully verifies the high-risk characteristics of the imperfect market. Due to the incomplete capital market, insufficient publicly available data, strict capital account control, imperfect market system and freedom not high, which leads to higher risk?

When the risk aversion coefficient γ decreases, the optimal investment proportion $\pi^*(t)$, the optimal expected value $E[X^*(t)]$ of ending wealth and the maximum expected utility value $E[U[X^*(t)]]$ of ending wealth of the SWFs investors will also decrease, while the optimal proportion of investing in risk-free assets will increase accordingly. This indicates that the change of investors' risk aversion will directly affect the investment strategy. The lower the risk acceptance rate is, the lower the investment proportion of risk assets will be, and the corresponding final utility wealth value and the maximum expected utility wealth value will be less. Therefore, when the risk tolerance changes, investors' investment strategies should be adjusted accordingly.

The average coefficient of liquidity index on stock assets is relatively high, which indicates that this index has a good prediction ability. Moreover, this index is negatively correlated with the return on equity assets, so when this index declines, the return on equity increases. In the incomplete market, if the investor does not consider the prediction variable and the power utility and asset income are independently and ideally distributed, the optimal allocation can be obtained indepent of the investment term. In this case, the investor shall allocate the stock asset in the same proportion regardless of the investment term. If the role of prediction variables is taken into account, the optimal allocation ratio of stock assets is higher than that without considering its prediction ability. The longer the investment period is, the higher the proportion will be. In other words, with the extension of the investment period, the proportion of stock assets owned by investors will increase. Therefore, the allocation decision of long-term investors is obviously different from that of shortterm investors. The longer the investment term is, the higher the allocation proportion of risk assets such as stocks will be.

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